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# MATHEMATICAL SIMULATION OF THE PROCESS OF COOLING AND FREEZING OF BODIES WITH VARIABLE THERMOPHYSICAL CHARACTERISTICS

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*A model and method are suggested for calculating the cooling and freezing of a rectangular block of fish as applied to rotory freezing units. A Fourier dimensionless heat-conduction equation is formulated with allowance for the variable character of the thermophysical properties of the object to be frozen. The presented results of the calculation carried out by the finite-difference method are in agreement with the experimental data.*

Freezing of foodstuffs is a complex thermophysical process. In refrigeration technology, the most important parameter is the freezing time $\tau_{fr}$, which is usually taken to mean the total time of cooling and freezing of products to a prescribed temperature. In determining the freezing time, the most widespread use is made of a Planck formula (see, for example, [1]), which is obtained for bodies of a simple shape with constancy of the thermophysical properties of the object and substantial assumptions. Subsequently, modifications of this formula obtained analytically with a smaller number of assumptions were suggested [1-4]. One of the latest solutions of this kind is given in [4], where allowance is made for the cooling time from the initial temperature to a cryoscopic one and of the duration of a phase transition and aftercooling of the frozen object. However, it was assumed in [4] that all the thermophysical properties of the object (heat capacity, thermal conductivity, specific heat of freezing) and the boundary conditions were constant (otherwise, the problem has no analytical solution).

In [5], the variable character of the thermophysical properties of the frozen object is considered and the posed problem is solved numerically. But here generalized relations for determining the freezing time do not contain the thermophysical characteristics of the product.

In the present work, we suggest a model and method for calculating the cooling and freezing of a rectangular block of fish as applied to rotory freezing units [6, 7], i.e., for a one-dimensional problem which is described by the Fourier equation [8]

$$\rho c \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial X}\left(\lambda \frac{\partial T}{\partial X}\right). \tag{1}$$

To Eq. (1) we prescribe the following boundary conditions:

• at the center of the block, the conditions of symmetry

$$\left(\frac{\partial T}{\partial X}\right)_{X=\delta/2} = 0\,, \tag{2}$$

• near the wall, the conditions of the third kind

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$$\left(-\lambda \frac{\partial T}{\partial X}\right)_{X=0} = \alpha (T_0 - T_f) . \quad (3)$$

The initial conditions are $T(X, 0) = T_i$.

Taking into account that fish in the block consists of dry substances with density $\rho_1$ and mass fraction $w_1$ as well as water with density $\rho_2$ and mass fraction $w_2 = 1 - w_1$, we take the mean density of the block as

$$\rho = \frac{\rho_1 \rho_2}{\rho_1 w_1 + \rho_2 w_2} \approx \text{const} . \quad (4)$$

The portion of water with mass fraction $w_{22}$ is nonfreezing (firmly bonded [1]); the other portion $w_{21}$ is in a solution and can be frozen: $w_{21} + w_{22} = w_2$.

The specific heat of the mixture at $T \geq T_{cry}$ ($T_{cry}$ is the cryoscopic temperature) is

$$c_{liq} = w_1 c_1 + w_2 c_{22} , \quad (5)$$

where $c_1$ is the specific heat of the dry substances and $c_{22}$ is the specific heat of the water in the liquid state.

To find the specific heat of the mixture at $T < T_{cry}$, we write the enthalpy increment

$$dH = \left\{ w_1 c_1 + w_{22} c_{22} + w_{21} \left[ (1 - \omega) c_{22} + \omega c_{21} \right] \right\} dT - w_{21} L d\omega . \quad (6)$$

Just as in [1], we calculate the amount of frozen-out water:

$$\omega = 1 - T_{cry}/T , \quad d\omega = (T_{cry}/T^2) \, dT . \quad (7)$$

Substituting Eq. (7) into Eq. (6), we obtain

$$c = \frac{dH}{dT} = c_{liq} - w_{21} \left[ \left( 1 - \frac{T_{cry}}{T} \right) (c_{22} - c_{21}) + \frac{L T_{cry}}{T^2} \right] .$$

Now we introduce the dimensionless heat capacity (at $T < T_{cry}$)

$$\tilde{c} \equiv \frac{c}{c_{liq}} = 1 - w_{21} \left[ \left( 1 - \frac{T_{cry}}{T} \right) \frac{c_{22} - c_{21}}{c_{liq}} + \frac{T_{cry}^2}{T^2} \frac{L}{c_{liq} T_{cry}} \right] = 1 - w_{21} \left[ \left( 1 - \frac{T_{cry}}{T} \right) \bar{c} + \frac{T_{cry}^2}{T^2} h \right] ,$$

$$\bar{c} = \frac{c_{22} - c_{21}}{c_{liq}} , \quad h = \frac{L}{c_{liq} T_{cry}} .$$

In order to put heat-conduction equation (1) into dimensionless form, we introduce the dimensionless temperature $\theta$ in the following manner [3]:

$$\theta = \frac{T - T_{cry}}{T_{cry} - T_f} . \quad (8)$$

It should be noted that $\theta = 0$ will correspond to the initial cryoscopic temperature, while $\theta = -1$ will correspond to the temperature of the refrigerant.

The thermal conductivity coefficient is not an additive characteristic and, according to [1], at $T < T_{cry}$ it can be determined from the empirical formula

$$\lambda = m + n/T . \quad (9)$$

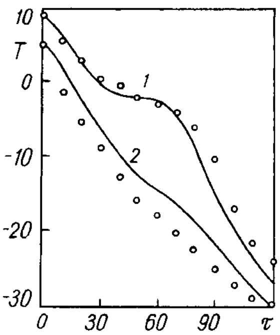



Fig. 1. Time variation in the temperature: 1) at the center of the block ; 2) at $X$ = 15 mm. $T$, °C; $\tau$, min.

When $T \ge T_{cry}$, $\lambda = \lambda_f$ = const. The dimensionless thermal conductivity coefficient $\tilde{\lambda} \equiv \lambda/\lambda_{fish}$ (when $T \ge T_{cry}$, $\tilde{\lambda} = 1$).

We substitute Eqs. (7)-(9) into Eq. (1) and after cancellation will have

$$\rho c_{liq} \tilde{c} \frac{\partial \theta}{\partial \tau} = \lambda_{fish} \frac{\partial}{\partial X}\left(\tilde{\lambda} \frac{\partial \theta}{\partial X}\right). \tag{10}$$

Multiplying both sides of Eq. (10) by the square of the block thickness $\delta^2$ and introducing the dimensionless coordinate $x$ and time $\tilde{\tau}$, we obtain the heat-conduction equation in dimensionless form:

$$\tilde{c} \frac{\partial \theta}{\partial \tilde{\tau}} = \frac{\partial}{\partial x}\left(\tilde{\lambda} \frac{\partial \theta}{\partial x}\right), \quad x = \frac{X}{\delta}, \quad \tilde{\tau} = \tau \frac{\lambda_{fish}}{\rho c_{liq} \delta^2}. \tag{11}$$

Although we were able to put the heat-conduction equation into a form not containing similarity numbers, they appear in closing relations (7) and (9) and in the dimensionless boundary conditions

$$\left(\frac{\partial \theta}{\partial x}\right)_{x=5} = 0; \quad \left(\tilde{\lambda} \frac{\partial \theta}{\partial x}\right)_{x=5} = \mathrm{Bi}\,(1 + \theta_0), \quad \mathrm{Bi} = \frac{\alpha \delta}{2\lambda_{fish}}. \tag{12}$$

Differential equation (11) with boundary conditions (12), as well as with the indicated initial conditions and closing relations, is solved numerically by the finite-difference method on an explicit four-point scheme [9]:

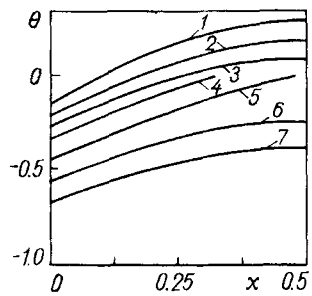
$$\tilde{c}_m^n \frac{\theta_m^{n+1} - \theta_m^n}{\Delta \tilde{\tau}} = \frac{1}{\Delta x^2}\left[\tilde{\lambda}_{m+1/2}^n \left(\theta_{m+1}^n - \theta_m^n\right) - \tilde{\lambda}_{m-1/2}^n \left(\theta_m^n - \theta_{m-1}^n\right)\right], \tag{13}$$

$$\tilde{\lambda}_{m+1/2}^n = 0.5\left(\tilde{\lambda}_{m+1}^n + \tilde{\lambda}_m^n\right), \quad \tilde{\lambda}_{m-1/2}^n = 0.5\left(\tilde{\lambda}_m^n + \tilde{\lambda}_{m-1}^n\right), \tag{14}$$

where $m$ = 0, 1, 2, ..., $M_1$; $M = M_1 - 1$, $M$ is the number of nodes of a difference-grid on half of the block thickness; $\Delta x$ is the difference-grid step along $x$, $x = 1/M$; $n$ = 0, 1, ... .

From Eq. (14) we can calculate the dimensionless temperature on the $(n + 1)$th layer using its value on the $n$th temporal layer beginning with $m$ = 1 to $M$:

$$\theta_m^{n+1} = \theta_m^n + \frac{\Delta \tilde{\tau}}{\Delta x^2 \tilde{c}_m^n}\left[\tilde{\lambda}_{m+1/2}^n \left(\theta_{m+1}^n - \theta_m^n\right) - \tilde{\lambda}_{m-1/2}^n \left(\theta_m^n - \theta_{m-1}^n\right)\right]. \tag{15}$$



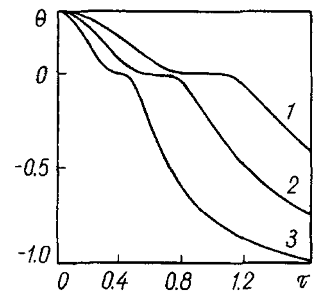




Fig. 2. Dependence of the dimensionless temperature profiles on time at Bi = 1: 1) $\tilde{\tau}$ = 0.2; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.0; 6) 1,4; 7) 1.6.

Fig. 3. Time variation in dimensionless temperatures at the center of the block: 1) Bi = 1; 2) 2; 3) 10.

The boundary conditions will be used to evaluate the values of the dimensionless temperature at the node on the block surface ($m = 0$) and at the center of the block ($m = M_1$):

$$\theta_0^{n+1} = \frac{\theta_1^{n+1} - \Delta x \, \mathrm{Bi}/\tilde{\lambda}_0^n}{1 + \Delta x \, \mathrm{Bi}/\tilde{\lambda}_0^n}; \quad \theta_{M_1}^{n+1} = \theta_M^{n+1} . \tag{16}$$

In view of the substantial nonlinearity of the problem, the number of nodes along $x$ must be rather large. In the present work, we used the grid with $M_1 = 100$ and time $\Delta\tilde{\tau}/\Delta x^2 = 0.1$. At larger values of the difference-grid steps the calculation results differed noticeably, i.e., the error in the numerical solution increased considerably.

To check the adequacy of the model, we compare the calculation results with the experimental data. Figure 1 presents the time variation in the temperature at the center of the block and at a distance of 15 mm from a freezing plate. The calculation of one variant took no more than 2 min of operation of the processor of a Pentium-200 computer. The experimental points [6] were obtained on an ARSA-3-15 pilot-industrial freezing installation when operated on Freon-22. The mass of the fish blocks was 11 kg, the mean thickness of the block was $\delta$ = 64 mm, and the Freon temperature, –40°C. According to [1], the values of the determining parameters for codfish were as follows: $w_2 = 0.803$; $w_{22} = 0.0682$; $T_{\mathrm{cry}} = -0.91$°C; the empirical coefficients in Eq. (9) for the codfish were $m = 1.23$ and $n = 0.58$. In the calculations, the initial temperature profile of the block was taken to be linear: from +1.5°C on the block edge to +9°C at the center of the block. From Fig. 1 it is seen that the calculated curves not only qualitatively but also satisfactorily quantitatively describe the dynamics of cooling and freezing of the fish block. The difference between the calculated curves and experimental points is probably attributable to the inaccuracy of representation of the variable thermal conductivity coefficient by means of empirical formula (9) and to the distinction of the boundary conditions from the theoretical ones, as well as to the one-dimensional model approximation and other factors.

Figures 2 and 3 present, as an example, the results of computer calculations of the cooling and freezing of the fish block at the values of the determining parameters indicated above. The temperature of the refrigerant as in an FGP-25-3 installation [5] was $T_f$ = –62°C, $T_i$ = 19°C; Bi = 1. The time variation in the profiles of the dimensionless temperature of the block is shown in Fig. 2. It can be seen that the region adjacent to the external boundary is quickly cooled to a temperature below the cryoscopic one ($\theta < 0$); at the center of the block, as the temperature attains the cryoscopic temperature, the process of water freezing continues for some time (curves 3 and 4). The time variation in the dimensionless temperature at the center of the block $\theta$ at three values of the Biot number is shown in Fig. 3. In all three cases, $\theta$ first attains zero (which in dimensional quantities corresponds to the cryoscopic temperature); after this it remains constant for some time and then continues to drop. It is seen that an increase in Bi intensifies this process. The results obtained agree with the physical meaning of the processes that occur in the cooling and freezing of foodstuffs.

Using the obtained dimensionless time of freezing to a prescribed temperature $\tilde{\tau}_3$ (Fig. 3), it is possible by formula (11) to calculate the dimensional quantity $\tau_3$ and the theoretical capacity of the freezing unit, proceeding from which one can determine the technological and actual productivity with allowance for cycle and extracycle time losses.

## NOTATION

$T$, local temperature, $T = T(x, \tau)$; $X$, coordinate along the block thickness $(0 \le X \le \delta)$; $\tau$, time; $\rho$, density; $c = c(T)$, specific heat; $\lambda = \lambda(T)$, coefficient of thermal conductivity; $\delta$, thickness of the fish block; $\alpha$, coefficient of heat transfer; $T_f$, temperature of the refrigerant; $T_0 = T(0, \tau)$, boundary conditions; $T(X, 0) = T_i$, initial conditions; $\rho_1$ and $\rho_2$, density of the dry substances and water, respectively; $w_1$, $w_2 = 1 - w_1$, their mass fractions; $L$, specific heat of water freezing; H, enthalpy; $\omega$, amount (portion) of the frozen-out water; $\mathrm{Bi} = \alpha\delta/2\lambda_{\mathrm{fish}}$, Biot number; $\theta$, dimensionless temperature. Subscripts: 1, dry substances; 2, water; 22, nonfreezing water; 21, water in the solution that can be frozen out; fish, fish.

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